

6.2D Polynomial Division – Part I

#1 – 5: Find each quotient using polynomial long division and state if the binomial is a factor.

1. $(2x^4 + 15x^3 - 30x^2 - 20x + 63) \div (x + 9)$

$$\begin{array}{r}
 2x^3 - 3x^2 - 3x + 7 \\
 x+9 \overline{) 2x^4 + 15x^3 - 30x^2 - 20x + 63} \\
 \underline{-(2x^4 + 18x^3)} \\
 -3x^3 - 30x^2 \\
 \underline{-(-3x^3 - 27x^2)} \\
 -3x^2 - 20x \\
 \underline{-(-3x^2 - 27x)} \\
 +7x + 63 \\
 \underline{-(7x + 63)} \\
 0
 \end{array}$$

yes, $(x+9)$ is a factor.

3. $(4x^2 - 5) \div (2x + 6)$

$$\begin{array}{r}
 2x - 6 + \frac{31}{2x+6} \\
 2x+6 \overline{) 4x^2 + 0x - 5} \\
 \underline{-(4x^2 + 12x)} \\
 -12x - 5 \\
 \underline{-(-12x - 36)} \\
 31
 \end{array}$$

not a factor

5. $(2x^3 - 5x^2 + 6x - 2) \div (2x - 1)$

$$\begin{array}{r}
 x^2 - 2x + 2 \\
 2x-1 \overline{) 2x^3 - 5x^2 + 6x - 2} \\
 \underline{-(2x^3 - x^2)} \\
 -4x^2 + 6x \\
 \underline{-(-4x^2 + 2x)} \\
 4x - 2 \\
 \underline{-(4x - 2)} \\
 0
 \end{array}$$

yes, $(2x-1)$ is a factor of $(2x^3 - 5x^2 + 6x - 2)$

2. $(5x^5 - 3x^4 + 2x^3 - 30x^2 - 7x + 3) \div (x - 2)$

$$\begin{array}{r}
 5x^4 + 7x^3 + 16x^2 + 2x - 3 - \frac{3}{x-2} \\
 x-2 \overline{) 5x^5 - 3x^4 + 2x^3 - 30x^2 - 7x + 3} \\
 \underline{-(5x^5 - 10x^4)} \\
 7x^4 + 2x^3 \\
 \underline{-(7x^4 - 14x^3)} \\
 16x^3 - 30x^2 \\
 \underline{-(16x^3 - 32x^2)} \\
 2x^2 - 7x \\
 \underline{-(2x^2 - 4x)} \\
 -3x + 3 \\
 \underline{-(-3x + 6)} \\
 -3
 \end{array}$$

not a factor

4. $(-4x^6 - 5x^3 + 3x^2 + x + 7) \div (x - 1)$

$$\begin{array}{r}
 -4x^5 - 4x^4 - 4x^3 - 9x^2 - 6x - 5 + \frac{2}{x-1} \\
 x-1 \overline{) -4x^6 + 0x^5 + 0x^4 - 5x^3 + 3x^2 + x + 7} \\
 \underline{-(4x^6 - 4x^5)} \\
 -4x^5 + 0x^4 \\
 \underline{-(-4x^5 + 4x^4)} \\
 -4x^4 - 5x^3 \\
 \underline{-(-4x^4 + 4x^3)} \\
 -9x^3 + 3x^2 \\
 \underline{-(-9x^3 + 9x^2)} \\
 -6x^2 + x \\
 \underline{-(-6x^2 + 6x)} \\
 -5x + 7 \\
 \underline{-(-5x + 5)} \\
 2
 \end{array}$$

not a factor

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#6 – 9: Find each quotient using synthetic division and state if the binomial is a factor.

6. $(x^3 + 6x^2 + 7x + 10) \div (x + 2)$

$$\begin{array}{r|rrrr} -2 & 1 & 6 & 7 & 10 \\ & & -2 & -8 & 2 \\ \hline & 1 & 4 & -1 & 12 \end{array}$$

Not a factor

$x^2 + 4x - 1 + \frac{12}{x+2}$

7. $\frac{4x^3 - 15x^2 - 120x - 128}{x - 8}$

$$\begin{array}{r|rrrr} 8 & 4 & -15 & -120 & -128 \\ & & 32 & 136 & 128 \\ \hline & 4 & 17 & 16 & 0 \end{array}$$

$4x^2 + 17x + 16$

$(x-8)$ is a factor of $(4x^3 - 15x^2 - 120x - 128)$.

8. $(3x^5 + 4x^3 - x - 2) \div (x - 1)$

$$\begin{array}{r|rrrrrr} 1 & 3 & 0 & 4 & 0 & -1 & -2 \\ & & 3 & 3 & 7 & 7 & 6 \\ \hline & 3 & 3 & 7 & 7 & 6 & 4 \end{array}$$

Not a factor

$3x^4 + 3x^3 + 7x^2 + 7x + 6 + \frac{4}{x-1}$

9. $\frac{x^3 - 3x^2 - 11x + 5}{x - 5}$

$$\begin{array}{r|rrrr} 5 & 1 & -3 & -11 & 5 \\ & & 5 & 10 & -5 \\ \hline & 1 & 2 & -1 & 0 \end{array}$$

$x^2 + 2x - 1$

$(x-5)$ is a factor of $(x^3 - 3x^2 - 11x + 5)$

10. Which of the division problems above (#1 – 9) generate no remainder? What does that mean regarding the relationship of the polynomials if no remainder occurs when dividing?

#7 and #9; The divisor is a factor of the dividend.

11. Lia, Maut and Craig were working on the following problems during class. Did they do the problems correctly? If not, explain what they did wrong and fix their mistakes.

Lia

$(x^3 + 2x^2 - 6x - 9) \div (x - 2)$

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -6 & -9 \\ & & 2 & 8 & 4 \\ \hline & 1 & 4 & 2 & -5 \end{array}$$

Correct, except to write the quotient:

$$x^2 + 4x + 2 - \frac{5}{x-2}$$

Maut

$\frac{10x^4 + 5x^3 + 4x^2 - 9}{x + 1}$

$$\begin{array}{r|rrrr} -1 & 10 & 5 & 4 & -9 \\ & & -10 & 5 & -9 \\ \hline & 10 & -5 & 9 & -18 \end{array}$$

missing a place holder for the x term.

$$\begin{array}{r|rrrrr} -1 & 10 & 5 & 4 & 0 & -9 \\ & & -10 & 5 & -9 & 9 \\ \hline & 10 & -5 & 9 & -9 & 0 \end{array}$$

$$10x^3 - 5x^2 + 9x - 9$$

Craig

$(x^2 - 4x + 3) \div (x - 2)$

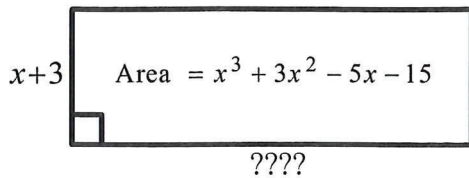
$$\begin{array}{r|rrr} -2 & 1 & -4 & 3 \\ & & -2 & 12 \\ \hline & 1 & -6 & 15 \end{array}$$

$$\begin{array}{r|rrr} 2 & 1 & -4 & 3 \\ & & 2 & -4 \\ \hline & 1 & -2 & -1 \end{array}$$

$$x - 2 - \frac{1}{x-2}$$

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12. Find the length of the rectangular garden.



$$\begin{array}{r} -3 \overline{) 1 \ 3 \ -5 \ -15} \\ \underline{-3 \ 0 \ 15} \\ 1 \ 0 \ -5 \ 0 \end{array}$$

$(x^2 - 5)$ is the length

$$W \cdot L = \text{Area}$$

$$(x+3)(????) = x^3 + 3x^2 - 5x - 15$$

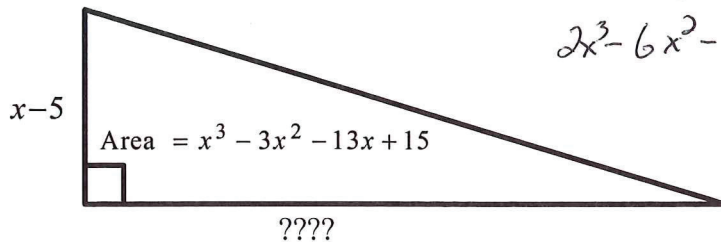
so if we divide $\frac{x^3 + 3x^2 - 5x - 15}{x+3}$

we can find the length (????)

13. Find the length of the base of the triangle below if
- $A = \frac{1}{2}bh$
- .

$$2A = bh$$

$$2x^3 - 6x^2 - 26x + 30 = (\text{base})(x-5)$$



$$\begin{array}{r} 5 \overline{) 2 \ -6 \ -26 \ 30} \\ \underline{10 \ 20 \ -30} \\ 2 \ 4 \ -6 \ 0 \end{array}$$

$(2x^2 + 4x - 6)$ is the Base

14. Suppose that you know that the area of a rectangular mural (wall painting) in square feet is represented by the polynomial
- $x^2 + 2x - 24$
- and that the length of the mural in feet if the length is represented by the binomial
- $x + 6$
- . How would you calculate the width of the mural? Would it also be a binomial?

Divide the area by the length (synthetically) to get the width.
Yes, the width would also be a binomial.

$$\begin{array}{r} -6 \overline{) 1 \ 2 \ -24} \\ \underline{-6 \ 24} \\ 1 \ -4 \ 0 \end{array}$$

$(x-4)$ is the width.

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15. If **A** and **B** are polynomials and **A** divided by **B** equals $5x^2 - 13x + 47 - \frac{102}{x+2}$.

a) Find **B**. $(x+2)$

b) Describe what you did to find this. *B is the divisor, located in the denominator of the remainder fraction.*

c) Find **A**.

$$\begin{aligned}\frac{A}{B} &= (5x^2 - 13x + 47) - \frac{102}{x+2} \\ B \cdot \frac{A}{B} &= B \cdot \left[(5x^2 - 13x + 47) - \frac{102}{x+2} \right] \\ A &= (x+2)(5x^2 - 13x + 47) - 102 \\ &= 5x^3 - 13x^2 + 47x + 10x^2 - 26x + 94 - 102 \\ &= 5x^3 - 3x^2 + 21x - 8\end{aligned}$$

16. Write a polynomial division problem where the use of synthetic division would be an appropriate strategy to use. Divide the polynomial problem you have written to find the quotient and remainder (if there is one).

one sample $(2x^3 - x + 3) \div (x-1)$

$$\begin{array}{r|rrrr} 1 & 2 & 0 & -1 & 3 \\ & & 2 & 2 & 1 \\ \hline & 2 & 2 & 1 & 4 \end{array}$$

$$\left\{ 2x^2 + 2x + 1 + \frac{4}{x-1} \right\}$$

17. Write a polynomial division problem which you cannot use synthetic division to simplify. Explain your reasoning why synthetic division cannot be used. Divide the polynomial expression you have written to find the quotient and remainder (if there is one).

one sample $(x^3 - 3x^2 + x - 3) \div (x^2 + 1)$

Using long division,

$$\begin{array}{r} x^2+1 \overline{) x^3 - 3x^2 + x - 3} \\ \underline{-(x^3 + x)} \\ -3x^2 - 3 \\ \underline{-(-3x^2 - 3)} \\ 0 \end{array}$$

($x-3$) is the quotient.

Synthetic Division can only be used when the divisor is a binomial with degree 1, like $(x-c)$

Section 6.2D